



**PICTURE FUZZY SOFT MATRICES USING A REVISED MAX-MIN-MIN
AVERAGE COMPOSITION METHOD FOR DECISION MAKING**

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Abstract:

In this paper, we extend the max-min-min average composition method, originally developed for intuitionistic fuzzy soft matrices, to Picture Fuzzy Soft Matrix (PFSM) theory. We formally propose the max-min-min Average Composition operator Φ for PFSMs and prove a closure theorem showing that the resulting composition satisfies the defining conditions of a picture fuzzy soft matrix. We apply the proposed operator through an enhanced multi-criteria decision-making model to a case study involving the selection of suitable job roles for applicants based on their professional skills.

1. Introduction:

Uncertainty forms an essential part of the nature of decisions in practical situations, especially those involving socio-economic, industrial, and environmental systems. Conventional deterministic approaches often fail to deal with situations involving vague, imprecise, and incomplete information. To deal with such situations, Zadeh [1] developed Fuzzy Set theory, where a membership value $\mu_A(x) \in [0,1]$ is used to describe the degree of belongingness of an element x to a set A . The soft set theory was then introduced by Molodtsov [2] for the purpose of solving the problem of parameterization and vagueness without resorting to predefined membership functions. Maji et al. [3, 4] combined these two ideas to establish Fuzzy Soft Sets and Intuitionistic Fuzzy Soft Sets (IFSS), embedding Atanassov's [5] Intuitionistic Fuzzy Set concepts into soft parameterization structures.

Matrix representation of fuzzy soft sets for efficient computation and representation was suggested by Yang and Ji [8]. T. JNeog and researchers presented fuzzy soft matrix theory and its decision making [9]. This was later extended to Intuitionistic Fuzzy Soft Matrix (IFSM) theory by Chetia and et al. [10, 11, 12] and by Rajarajeswari and Dhanalakshmi [13, 14]. Shanmugasundaram et al. [15, 16] presented max-min average and revised max-min average composition methods for IFSMs, applying them to medical diagnosis and decision-making problems. Their results showed that average-based matrix composition reduces decision ties and evaluation conflicts compared to classical max-min composition operators.

Though helpful, IFSS and IFSM models allow parametric assessment through a positive membership μ and a negative membership ν , subject to the intuitionistic fuzzy constraint $0 \leq \mu + \nu \leq 1$.

In practice, human decisions usually involve not two, but four possibilities: "yes," "abstain," "no," and "refusal." The above-described evaluation technique results from the analysis of voting, forecasting economic conditions, and solving multi-criteria problems of building infrastructure. To adapt the new type of evaluation process, Cuong [17] introduced Picture Fuzzy Sets (PFS), characterized by a positive membership degree μ , a neutral membership degree η , and a negative membership degree ν , subject to $0 \leq \mu + \eta + \nu \leq 1$. The remaining quantity $\pi = 1 - \mu - \eta - \nu$ represents the degree of refusal.

The matrix representation of picture fuzzy information and its applications have been studied by several researchers [18, 21]. The combination of picture fuzzy sets with soft sets results in the formation of Picture Fuzzy Soft Sets (PFSS), which improves parametric modeling. The representation of PFSS using matrices called Picture Fuzzy Soft Matrices (PFSMs) has been studied [20, 19] although composition operations for PFSMs require further investigation with respect to algebraic closure and interaction among the three membership degrees. This paper addresses these gaps through four main contributions:

- The max-min average composition method is extended to Picture Fuzzy Soft Matrices (PFSMs) by proposing a Max-Min-Min Average Composition operator.
- We state and prove the Closure Theorem for PFSMs under the proposed Max-Min-Min average composition operator Φ , showing that the resulting matrices satisfy the fuzzy boundary conditions required for the picture.

- We propose an extended score matrix that includes positive, neutral, and negative membership degrees to obtain a well-defined ranking technique.
- The proposed model is applied to a case study involving candidates with professional expertise, demonstrating its application using numerical examples in a multi-criteria decision-making environment.

2. Preliminaries:

This section reviews the foundational definitions of soft sets, intuitionistic fuzzy soft sets, picture fuzzy sets, picture fuzzy soft sets, and their matrix representations.

Definition 1: (Soft Set). [2]

Let U be a universe of discourse and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U . A pair (F, A) is called a soft set over U , where $A \subseteq E$ and F is a mapping given by $F: A \rightarrow \mathcal{P}(U)$. A soft set is a parameterized family of subsets of the universe U .

Definition 2: (Fuzzy Soft Set). [3]

Let U be the universe of discourse, E be the set of parameters, and $A \subseteq E$. Let I^U denote the collection of all fuzzy subsets of U . A pair (F, A) is called a fuzzy soft set over U , if $F: A \rightarrow I^U$.

Definition 3: (Intuitionistic Fuzzy Soft Set). [4]

Let U be a universe of discourse, E a set of parameters, and $A \subseteq E$. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U . A pair (F, A) is called an intuitionistic fuzzy soft set over U , if $F: A \rightarrow IF^U$. Each parameter $e \in A$ maps to an intuitionistic fuzzy set $F(e) = \{(c_i, \mu_{F(e)}(c_i), \nu_{F(e)}(c_i)) : c_i \in U\}$, where $\mu_{F(e)}(c_i) \in [0,1]$ and $\nu_{F(e)}(c_i) \in [0,1]$ satisfy $\mu_{F(e)}(c_i) + \nu_{F(e)}(c_i) \leq 1$.

Definition 4: (Picture Fuzzy Set). [17]

Let U be a universe of discourse. A Picture Fuzzy Set (PFS) P on U is defined as $P = \{(\mu_P(x), \eta_P(x), \nu_P(x)) : x \in U\}$, where $\mu_P, \eta_P, \nu_P: U \rightarrow [0,1]$ are the positive membership, neutral membership, and negative membership degrees, respectively, satisfying the condition $0 \leq \mu_P(x) + \eta_P(x) + \nu_P(x) \leq 1, \forall x \in U$. For each element $x \in U$, the refusal degree $\pi_P(x)$ is defined as $\pi_P(x) = 1 - (\mu_P(x) + \eta_P(x) + \nu_P(x))$.

Definition 5: (Picture Fuzzy Soft Set). [17]

Let $U = \{c_1, c_2, \dots, c_m\}$ be the universal set of objects and $E = \{e_1, e_2, \dots, e_n\}$ be the parameter set. Let PFS^U denote the collection of all picture fuzzy subsets of U . A pair (F, A) , where $A \subseteq E$, is called a Picture Fuzzy Soft Set (PFSS) over U if $F: A \rightarrow PFS^U$.

Definition 6: (Picture Fuzzy Soft Matrix). [19]

Let (F, A) be a Picture Fuzzy Soft Set over U . Then (F, A) has the matrix representation

$$A_{m \times n} = [a_{ij}]_{m \times n},$$

Where

$$a_{ij} = \begin{cases} (\mu_j(c_i), \eta_j(c_i), \nu_j(c_i)), & \text{if } e_j \in A, \\ (0, 0, 1), & \text{if } e_j \notin A, \end{cases}$$

Where $\mu_j(c_i)$, $\eta_j(c_i)$, and $\nu_j(c_i)$ represent the positive, neutral, and negative membership values of element c_i with respect to parameter e_j , respectively, satisfying the condition $0 \leq \mu_j(c_i) + \eta_j(c_i) + \nu_j(c_i) \leq 1$. The set of all $m \times n$ Picture Fuzzy Soft Matrices over U is denoted by $PFSM_{m \times n}$.

3. Picture Fuzzy Soft Matrix Operations:

Definition 7: [19, 20]

Let $A = [a_{ij}]_{m \times n} = [(\mu_{A,ij}, \eta_{A,ij}, \nu_{A,ij})] \in PFSM_{m \times n}$ and $B = [b_{ij}]_{m \times n} = [(\mu_{B,ij}, \eta_{B,ij}, \nu_{B,ij})] \in PFSM_{m \times n}$.

The sum, difference, and complement operators are defined as

- $A + B = [(\max[\mu_{A,ij}, \mu_{B,ij}], \min[\eta_{A,ij}, \eta_{B,ij}], \min[\nu_{A,ij}, \nu_{B,ij}])]$,
- $A - B = [(\min[\mu_{A,ij}, \mu_{B,ij}], \min[\eta_{A,ij}, \eta_{B,ij}], \max[\nu_{A,ij}, \nu_{B,ij}])]$, and
- $A^c = [(\nu_{A,ij}, \eta_{A,ij}, \mu_{A,ij})]$.

Definition 8: [19, 20]

For $A = [a_{ij}]_{m \times n} \in PFSM_{m \times n}$ and $B = [b_{jk}]_{n \times p} \in PFSM_{n \times p}$, the classical max-min composition, denoted by $A * B = [c_{ik}]_{m \times p}$,

is defined component-wise as $c_{ik} = \left(\max_{1 \leq j \leq n} \{ \min[\mu_{A,ij}, \mu_{B,jk}] \}, \min_{1 \leq j \leq n} \{ \max[\eta_{A,ij}, \eta_{B,jk}] \}, \min_{1 \leq j \leq n} \{ \max[\nu_{A,ij}, \nu_{B,jk}] \} \right)$.

Definition 9: Max-Min-Min Average Composition Operator (Proposed)

Let $A = [a_{ij}]_{m \times n} \in PFSM_{m \times n}$ and $B = [b_{jk}]_{n \times p} \in PFSM_{n \times p}$. The proposed Max-Min-Min Average Composition of A and B , denoted by $A \Phi B = [c_{ik}]_{m \times p}$, is defined component-wise as

$$c_{ik} = \left(\max_{1 \leq j \leq n} \left\{ \frac{\mu_{A,ij} + \mu_{B,jk}}{2} \right\}, \min_{1 \leq j \leq n} \left\{ \frac{\eta_{A,ij} + \eta_{B,jk}}{2} \right\}, \min_{1 \leq j \leq n} \left\{ \frac{\nu_{A,ij} + \nu_{B,jk}}{2} \right\} \right),$$

for all $i = 1, 2, \dots, m$ and $k = 1, 2, \dots, p$.

Definition 10: Score Matrix (Proposed).

Let A^c and B^c denote the complements of A and B , respectively. The score matrix of A and B , denoted by $S(A, B)$, is defined as $S(A, B) = \frac{1}{2}(V_s + W_s)$, Where V_s and W_s are the $m \times p$ matrices whose (i, k) -th

entries are given by $(V_s)_{ik} = [(\mu_A \Phi \mu_B)_{ik} + (\eta_A \Phi \eta_B)_{ik}] - (v_A^c \Phi v_B^c)_{ik}$, and $(W_s)_{ik} = [(\mu_A^c \Phi \mu_B^c)_{ik} + (\eta_A^c \Phi \eta_B^c)_{ik}] - (v_A \Phi v_B)_{ik}$, for all $i = 1, 2, \dots, m$ and $k = 1, 2, \dots, p$.

Example 1:

Let

$$A = \begin{bmatrix} (0.6, 0.3, 0.1) & (0.4, 0.3, 0.2) \\ (0.5, 0.2, 0.3) & (0.3, 0.4, 0.2) \end{bmatrix} \in PFSM_{2 \times 2}$$

and

$$B = \begin{bmatrix} (0.5, 0.1, 0.2) & (0.7, 0.2, 0.1) \\ (0.4, 0.3, 0.2) & (0.6, 0.2, 0.1) \end{bmatrix} \in PFSM_{2 \times 2}.$$

then,

$$A + B = \begin{bmatrix} (0.6, 0.1, 0.1) & (0.7, 0.2, 0.1) \\ (0.5, 0.2, 0.2) & (0.6, 0.2, 0.1) \end{bmatrix} \in PFSM_{2 \times 2},$$

$$A - B = \begin{bmatrix} (0.5, 0.1, 0.2) & (0.4, 0.2, 0.2) \\ (0.4, 0.2, 0.3) & (0.3, 0.2, 0.2) \end{bmatrix} \in PFSM_{2 \times 2},$$

$$A^c = \begin{bmatrix} (0.1, 0.3, 0.6) & (0.2, 0.3, 0.4) \\ (0.3, 0.2, 0.5) & (0.2, 0.4, 0.3) \end{bmatrix} \in PFSM_{2 \times 2},$$

$$A * B = \begin{bmatrix} (0.5, 0.3, 0.2) & (0.6, 0.3, 0.1) \\ (0.5, 0.2, 0.2) & (0.5, 0.2, 0.2) \end{bmatrix} \in PFSM_{2 \times 2}.$$

The revised max-min average composition of A and B is

$$A \Phi B = \begin{bmatrix} (0.55, 0.20, 0.15) & (0.65, 0.25, 0.10) \\ (0.50, 0.15, 0.20) & (0.60, 0.20, 0.15) \end{bmatrix} \in PFSM_{2 \times 2}.$$

4. Main Results: Closure Theorem and Algebraic Properties

In this section, we establish the closure theorem for the Revised Max-Min-Min Average Composition operator Φ on Picture Fuzzy Soft Matrices.

Theorem 1: (Closure Theorem).

Let $A = [a_{ij}]_{m \times n} \in PFSM_{m \times n}$ and $B = [b_{jk}]_{n \times p} \in PFSM_{n \times p}$ be Picture Fuzzy Soft Matrices. Then the revised max-min-min average composition matrix $C = A \Phi B = [c_{ik}]_{m \times p}$ belongs to $PFSM_{m \times p}$.

Proof:

$$\text{Let } A = [(\mu_{A,ij}, \eta_{A,ij}, v_{A,ij})]_{m \times n} \text{ and } B = [(\mu_{B,jk}, \eta_{B,jk}, v_{B,jk})]_{n \times p}.$$

Since A and B are Picture Fuzzy Soft Matrices, their components satisfy

$$0 \leq \mu_{A,ij}, \eta_{A,ij}, v_{A,ij} \leq 1, \quad \mu_{A,ij} + \eta_{A,ij} + v_{A,ij} \leq 1,$$

and

$$0 \leq \mu_{B,jk}, \eta_{B,jk}, v_{B,jk} \leq 1, \quad \mu_{B,jk} + \eta_{B,jk} + v_{B,jk} \leq 1,$$

for all admissible indices i, j, k .

By Definition 9, the components of $C = A \Phi B$

For any fixed pair (i, k) are given by

$$\mu_{C,ik} = \max_{1 \leq j \leq n} \left\{ \frac{\mu_{A,ij} + \mu_{B,jk}}{2} \right\},$$

$$\eta_{C,ik} = \min_{1 \leq j \leq n} \left\{ \frac{\eta_{A,ij} + \eta_{B,jk}}{2} \right\},$$

and

$$v_{C,ik} = \min_{1 \leq j \leq n} \left\{ \frac{v_{A,ij} + v_{B,jk}}{2} \right\}.$$

For any $j \in \{1, 2, \dots, n\}$, since

$$0 \leq \mu_{A,ij}, \mu_{B,jk} \leq 1,$$

We have

$$0 \leq \frac{\mu_{A,ij} + \mu_{B,jk}}{2} \leq 1.$$

Therefore,

$$0 \leq \mu_{C,ik} \leq 1.$$

Similarly,

$$0 \leq \frac{\eta_{A,ij} + \eta_{B,jk}}{2} \leq 1,$$

and hence

$$0 \leq \eta_{C,ik} \leq 1.$$

Likewise,

$$0 \leq \frac{v_{A,ij} + v_{B,jk}}{2} \leq 1,$$

which gives

$$0 \leq v_{C,ik} \leq 1.$$

It remains to show that

$$\mu_{C,ik} + \eta_{C,ik} + v_{C,ik} \leq 1.$$

Let $j^* \in \{1, 2, \dots, n\}$ be an index at which the maximum defining the positive membership component is attained. Thus,

$$\mu_{C,ik} = \frac{\mu_{A,ij^*} + \mu_{B,j^*k}}{2}.$$

Since $\eta_{C,ik}$ and $\nu_{C,ik}$ are defined as minima over j , their values are bounded above by their corresponding values at $j = j^*$. Hence,

$$\eta_{C,ik} \leq \frac{\eta_{A,ij^*} + \eta_{B,j^*k}}{2},$$

and

$$\nu_{C,ik} \leq \frac{\nu_{A,ij^*} + \nu_{B,j^*k}}{2}.$$

Since A and B are Picture Fuzzy Soft Matrices,

$$\mu_{A,ij^*} + \eta_{A,ij^*} + \nu_{A,ij^*} \leq 1$$

and

$$\mu_{B,j^*k} + \eta_{B,j^*k} + \nu_{B,j^*k} \leq 1.$$

Therefore,

$$\begin{aligned} \mu_{C,ik} + \eta_{C,ik} + \nu_{C,ik} &\leq \frac{\mu_{A,ij^*} + \mu_{B,j^*k}}{2} + \frac{\eta_{A,ij^*} + \eta_{B,j^*k}}{2} + \frac{\nu_{A,ij^*} + \nu_{B,j^*k}}{2} \\ &= \frac{(\mu_{A,ij^*} + \eta_{A,ij^*} + \nu_{A,ij^*}) + (\mu_{B,j^*k} + \eta_{B,j^*k} + \nu_{B,j^*k})}{2} \\ &\leq \frac{1 + 1}{2} = 1. \end{aligned}$$

Since

$$\mu_{C,ik}, \eta_{C,ik}, \nu_{C,ik} \geq 0,$$

We also have

$$0 \leq \mu_{C,ik} + \eta_{C,ik} + \nu_{C,ik}.$$

Consequently,

$$0 \leq \mu_{C,ik} + \eta_{C,ik} + \nu_{C,ik} \leq 1$$

for all

$$i \in \{1, \dots, m\}, \quad k \in \{1, \dots, p\}.$$

Thus,

$$C = A \Phi B \in PFSM_{m \times p}.$$

Therefore, the Revised Max-Min-Min Average Composition operator Φ is closed on the class of Picture Fuzzy Soft Matrices.

Proposition 2: (Involution of the Complement Matrix).

Let $A = [a_{ij}]_{m \times n} \in PFSM_{m \times n}$. Then $(A^c)^c = A$.

Proof:

By Definition 7, the complement of A is given by $A^c = [(v_{A,ij}, \eta_{A,ij}, \mu_{A,ij})]_{m \times n}$.

Applying the complement operation again, we obtain $(A^c)^c = [(\mu_{A,ij}, \eta_{A,ij}, \nu_{A,ij})]_{m \times n}$.

Hence, $(A^c)^c = A$.

Therefore, the complement operation on Picture Fuzzy Soft Matrices is involutive.

5. Numerical Example: Graduate Applicant-Skill-Job Position Selection Using Picture Fuzzy Soft Matrices

In this section, the proposed revised max-min-min average composition method and the picture fuzzy score function are applied to a graduate applicant-skill-job position selection problem. The objective is to evaluate the suitability of graduate applicants with respect to a set of professional skills and subsequently identify the job position for which each applicant has the highest suitability. The proposed picture fuzzy soft matrix framework enables recruitment experts to express positive, neutral, and negative assessments simultaneously. This is particularly suitable for recruitment and placement situations, where an evaluator may positively recognize an applicant's competence, remain neutral about a particular skill, or express a negative assessment.

The decision-making algorithm is summarized as follows.

- Step 1: Construct the PFSMs A and B from the given picture fuzzy soft sets.
- Step 2: Determine the complements A^c and B^c .
- Step 3: Compute the compositions $A \Phi B$ and $A^c \Phi B^c$.
- Step 4: Determine the score matrices V_s and W_s and subsequently calculate the final score matrix $S(A, B)$.
- Step 5: For each applicant, the largest entry in the corresponding row of $S(A, B)$ identifies the recommended job position.

Let $G = \{G_1, G_2, G_3, G_4\} = \{\text{Arjun, Meera, Kiran, Devan}\}$ be the set of four graduate applicants, $P = \{P_1, P_2, P_3, P_4\} = \{\text{Technical Skill, Analytical Skill, Leadership Skill, Communication Skill}\}$ be the set of four professional skills,

And $J = \{J_1, J_2, J_3, J_4\} = \{\text{Software Developer, Business Analyst, Project Manager, Data Scientist}\}$ be the set of four available job positions.

Step 1: Construction of the PFSMs

Consider the applicant-skill PFSM

$$A = \begin{bmatrix} (0.55, 0.15, 0.10) & (0.40, 0.20, 0.15) & (0.35, 0.10, 0.40) & (0.50, 0.15, 0.10) \\ (0.30, 0.15, 0.40) & (0.45, 0.10, 0.30) & (0.55, 0.15, 0.15) & (0.40, 0.20, 0.20) \\ (0.45, 0.10, 0.30) & (0.35, 0.15, 0.35) & (0.30, 0.10, 0.45) & (0.55, 0.10, 0.20) \\ (0.50, 0.15, 0.15) & (0.55, 0.10, 0.20) & (0.40, 0.15, 0.30) & (0.35, 0.20, 0.30) \end{bmatrix}$$

and the skill-job position PFSM

$$B = \begin{bmatrix} (0.55, 0.10, 0.20) & (0.40, 0.15, 0.30) & (0.30, 0.10, 0.45) & (0.45, 0.15, 0.25) \\ (0.45, 0.15, 0.25) & (0.55, 0.10, 0.20) & (0.35, 0.15, 0.35) & (0.40, 0.10, 0.35) \\ (0.30, 0.15, 0.40) & (0.35, 0.10, 0.40) & (0.55, 0.15, 0.15) & (0.30, 0.20, 0.35) \\ (0.50, 0.10, 0.25) & (0.45, 0.15, 0.20) & (0.40, 0.10, 0.35) & (0.55, 0.10, 0.20) \end{bmatrix}$$

Step 2: Complement PFSMs

The complement of A is

$$A^c = \begin{bmatrix} (0.10, 0.15, 0.55) & (0.15, 0.20, 0.40) & (0.40, 0.10, 0.35) & (0.10, 0.15, 0.50) \\ (0.40, 0.15, 0.30) & (0.30, 0.10, 0.45) & (0.15, 0.15, 0.55) & (0.20, 0.20, 0.40) \\ (0.30, 0.10, 0.45) & (0.35, 0.15, 0.35) & (0.45, 0.10, 0.30) & (0.20, 0.10, 0.55) \\ (0.15, 0.15, 0.50) & (0.20, 0.10, 0.55) & (0.30, 0.15, 0.40) & (0.30, 0.20, 0.35) \end{bmatrix}$$

The complement of B is

$$B^c = \begin{bmatrix} (0.20, 0.10, 0.55) & (0.30, 0.15, 0.40) & (0.45, 0.10, 0.30) & (0.25, 0.15, 0.45) \\ (0.25, 0.15, 0.45) & (0.20, 0.10, 0.55) & (0.35, 0.15, 0.35) & (0.35, 0.10, 0.40) \\ (0.40, 0.15, 0.30) & (0.40, 0.10, 0.35) & (0.15, 0.15, 0.55) & (0.35, 0.20, 0.30) \\ (0.25, 0.10, 0.50) & (0.20, 0.15, 0.45) & (0.35, 0.10, 0.40) & (0.20, 0.10, 0.55) \end{bmatrix}$$

Step 3: Revised Max-Min-Min Average Composition

The revised max-min-min average composition of A and B is $A \Phi B$

$$= \begin{bmatrix} (0.550, 0.125, 0.150) & (0.475, 0.100, 0.150) & (0.450, 0.125, 0.225) & (0.525, 0.125, 0.150) \\ (0.450, 0.125, 0.225) & (0.500, 0.100, 0.200) & (0.550, 0.125, 0.150) & (0.475, 0.100, 0.200) \\ (0.525, 0.100, 0.225) & (0.500, 0.100, 0.200) & (0.475, 0.100, 0.275) & (0.550, 0.100, 0.200) \\ (0.525, 0.125, 0.175) & (0.550, 0.100, 0.200) & (0.475, 0.125, 0.225) & (0.475, 0.100, 0.200) \end{bmatrix}$$

Similarly, $A^c \Phi B^c$

$$= \begin{bmatrix} (0.400, 0.120, 0.320) & (0.400, 0.100, 0.350) & (0.270, 0.120, 0.370) & (0.370, 0.120, 0.320) \\ (0.300, 0.120, 0.420) & (0.350, 0.100, 0.350) & (0.420, 0.120, 0.300) & (0.320, 0.100, 0.370) \\ (0.420, 0.100, 0.300) & (0.420, 0.100, 0.320) & (0.370, 0.100, 0.350) & (0.400, 0.100, 0.300) \\ (0.350, 0.120, 0.350) & (0.350, 0.100, 0.370) & (0.320, 0.120, 0.370) & (0.320, 0.100, 0.350) \end{bmatrix}$$

Step 4: Calculation of Score Matrix

The score matrices are

$$V_s = \begin{bmatrix} 0.3500 & 0.2250 & 0.2000 & 0.3250 \\ 0.1500 & 0.2500 & 0.3750 & 0.2000 \\ 0.3250 & 0.2750 & 0.2250 & 0.3500 \\ 0.3000 & 0.2750 & 0.2250 & 0.2250 \end{bmatrix}, \quad W_s = \begin{bmatrix} 0.3750 & 0.3500 & 0.1750 & 0.3500 \\ 0.2000 & 0.2500 & 0.4000 & 0.2250 \\ 0.3000 & 0.3250 & 0.2000 & 0.3000 \\ 0.3000 & 0.2500 & 0.2250 & 0.2250 \end{bmatrix}$$

Hence, the picture fuzzy score matrix is

$$S(A, B) = \frac{1}{2} [V_s + W_s] = \begin{bmatrix} 0.36250 & 0.28750 & 0.18750 & 0.33750 \\ 0.17500 & 0.25000 & 0.38750 & 0.21250 \\ 0.31250 & 0.30000 & 0.21250 & 0.32500 \\ 0.30000 & 0.26250 & 0.22500 & 0.22500 \end{bmatrix}$$

Step 5: Selection of Job Position

The final score matrix obtained in Step 4 is

$$S(A, B) = \begin{bmatrix} 0.3625 & 0.2870 & 0.1870 & 0.3370 \\ 0.1750 & 0.2500 & 0.3870 & 0.2120 \\ 0.3125 & 0.3000 & 0.2120 & 0.3250 \\ 0.3000 & 0.2620 & 0.2250 & 0.2250 \end{bmatrix}$$

For each graduate applicant, the maximum score in the corresponding row of $S(A, B)$ determines the recommended job position. The selected positions are given below:

G_1 (Arjun) $\rightarrow J_1$ (Software Developer),

G_2 (Meera) $\rightarrow J_3$ (Project Manager),

G_3 (Kiran) $\rightarrow J_4$ (Data Scientist),

G_4 (Devan) $\rightarrow J_1$ (Software Developer).

Therefore, the maximum score for Arjun is 0.3625 which shows the score for the Software Developer position; The maximum score for Meera is 0.3875 which highlights the score for the Project

Manager position; The maximum score for Kiran is 0.3250, which suits the score for the Data Scientist position and the maximum score, for Devan is 0.3000 corresponding to the Software Developer position.

6. Conclusion:

The proposed method offered a useful structure for decision making through PFSMs. The closure theorem established that the resulting composition remained a PFSM. Its application for job role selection produced rankings that were meaningful for both applicants and their roles. Further investigations might extend the suggested approach to Complex Picture Fuzzy Soft Matrices (CPFSMs) and other generalized fuzzy soft types.

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